SEPARABLE REDUCTIONS, RICH FAMILIES AND PROJECTIONAL SKELETONS IN NON-SEPARABLE BANACH SPACES

MARIÁN FABIAN, MATHEMATICAL INSTITUTE, CZECH ACADEMY OF SCIENCES, PRAHA

A Banach space X is isomorphic to a Hilbert space if (and only if) every separable subspace $Y \subset X$ is such. Similarly, X is reflexive if (and only if) every separable subspace of it is reflexive. The Radon Nikodým property (which is equivalent to dentability) is separably reducible. The Asplund property of a space is also separably reducible, ...

We will also separably reduce properties of functions, like boundedness, continuity, (sub)differentiability, ... E.g. a function $f : X \to \mathbb{R}$, there is a cofinal family $\mathcal{S}(X)$ of separable subspaces of X such that for every $x \in Y \in \mathcal{S}(X)$: f is continuous at x if (and only if) $f|_Y$ is continuous at x. By strengthening this, we arrive at the concept of *rich family* — meaning a family which is cofinal and in addition σ-closed. Rich families are very useful in grasping non-separable Banach spaces. They can serve as an index set for the so called projectional skeletons (PS), introduced in 2009 by W. Kubiś. PS is a modern substitute of the classical Lindenstrauss' projectional resolution of identity (PRI). Of course, from PS one can construct PRI. Several big classes of non-separable Banach spaces can be characterized by presence of a PS with some extra properties.

Separable reductions, rich families, and projectional skeletons in non-separable Banach spaces

Marian Fabian, Czech Academy of Sciences, Praha

A Banach space X is reflexive, dentable, Asplund, or isomorphic to a Hilbert space if and only if every separable subspace of it is, respectively, such. Further we also separably reduce properties of functions like continuity, differentiability etc. In the course of this, there naturally appear the so called rich families consisting of some separable subspaces of X. These families also serve as an index set for the so called projectional skeletons (PS) a modern substitute of the classical Lindenstrauss' projectional resolution of identity (W. Kubiś 2009). PS are then used for caracterizing various classes of Banach spaces like spaces reflexive, WCG, Asplund, Pličko, etc.