# Extensions of KKM theorem: $L^*-$ operators, $L^*-$ operators, and Colorful KKM.

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An $L^*-$*operator* on a topological space $X$ is any multifunction $\Lambda : [X]^{<\omega} \to \exp(X)$ satisfying the following condition: $(*)$ If $A \in [X]^{<\omega}$ and $\{U_x : x \in A\}$ is an open cover of $X,$ then there exists a $B \subseteq A$ such that $\Lambda(B) \cap \bigcap \{U_x : x \in B\} \neq \emptyset$. The convex hull operator on any topological vector space is an $L^*-$*operator* ( in fact, it is a different version of the celebrated KKM theorem in disguise ). The convex hull operator is also $n-$continuous for each $n = 1, 2, \ldots$ . We show, among others, that any metrizable continuum admitting a $2-$continuous $L^*-$*operator* must be locally connected and unicoherent. If it is also one-dimensional, then it has the fixed point property.

$L^*-$spaces are convexity structures induced by $L^*-$operators. We analyze some relationships between K-simplicial structures, $L-$structures, and $L^*-$spaces. A question due to H. Ben-El-Mechaiekh is answered by showing that K-simplicial structures coincide with $L-$structures provided they are $T_1$. We construct new examples distinguishing $L^*-$structures from $L-$structures and we also characterize absolute retracts in terms of $L-$structures.

Colorful KKM is a colorful version of KKM theorem. We show it holds true for families of corner simplices.

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